

# COMMON CORE STATE STANDARDS FOR

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## Mathematics



# Introduction

## Toward greater focus and coherence

*Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.*

— Mathematics Learning in Early Childhood, National Research Council, 2009

*The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.*

— Ginsburg, Leinwand and Decker, 2009

*Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.*

— Ginsburg et al., 2005

*There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.*

— Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge.

It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

*articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, **but also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies*

*that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties).* (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the “sequence of topics and performances” that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing “sequenced obstacles and challenges for students...absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.” In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time.

### **Understanding mathematics**

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as  $(a + b)(x + y)$  and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding  $(a + b + c)(x + y)$ . Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

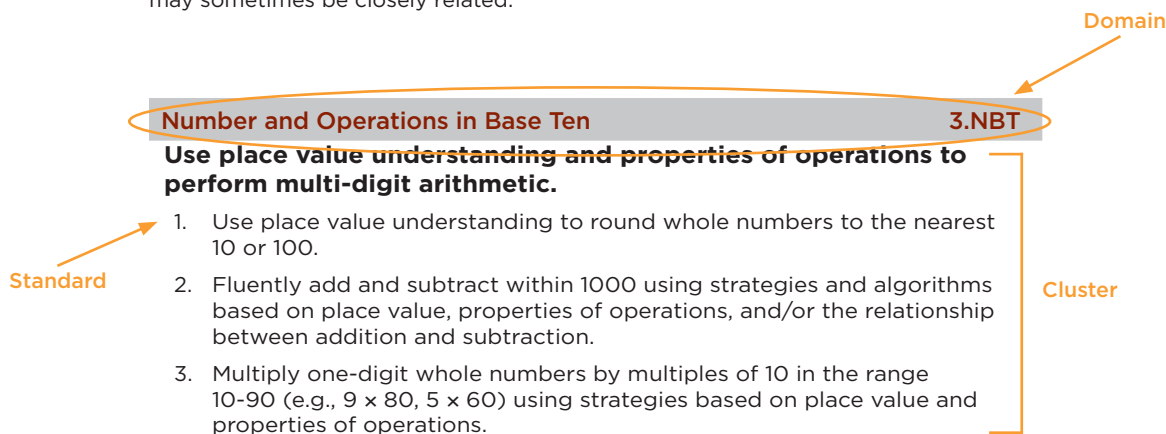
The Standards begin on page 6 with eight Standards for Mathematical Practice.

## How to read the grade level standards

**Standards** define what students should understand and be able to do.

**Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

**Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.



These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

# Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

#### **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

# Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

**Critical Area 1:** By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

**Critical Area 2:** In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

**Critical Area 3:** By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

**Critical Area 4:** This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

**Critical Area 5:** In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

**Critical Area 6:** Building on their work with the Pythagorean Theorem in 8<sup>th</sup> grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.



Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> Relationships Between Quantities	<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> <li>Interpret the structure of expressions.</li> <li>Create equations that describe numbers or relationships.</li> </ul>	<b>Make sense of problems and persevere in solving them.</b>  <b>Reason abstractly and quantitatively.</b>  <b>Construct viable arguments and critique the reasoning of others.</b>  <b>Model with mathematics.</b>  <b>Use appropriate tools strategically.</b>  <b>Attend to precision.</b>  <b>Look for and make use of structure.</b>  <b>Look for and express regularity in repeated reasoning.</b>
<b>Unit 2</b> Linear and Exponential Relationships	<ul style="list-style-type: none"> <li>Represent and solve equations and inequalities graphically.</li> <li>Understand the concept of a function and use function notation.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul>	
<b>Unit 3†</b> Reasoning with Equations	<ul style="list-style-type: none"> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Solve equations and inequalities in one variable.</li> <li>Solve systems of equations.</li> </ul>	
<b>Unit 4</b> Descriptive Statistics	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> <li>Interpret linear models.</li> </ul>	
<b>Unit 5</b> Congruence, Proof, and Constructions	<ul style="list-style-type: none"> <li>Experiment with transformations in the plane.</li> <li>Understand congruence in terms of rigid motions.</li> <li>Make geometric constructions.</li> </ul>	
<b>Unit 6</b> Connecting Algebra and Geometry through Coordinates	<ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

†Note that solving equations and systems of equations follows a study of functions in this course. To examine equations before functions, this unit could be merged with Unit 1.

## Unit 1: Relationships Between Quantities

By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Unit 1: Relationships between Quantities	
Clusters with Instructional Notes	Common Core State Standards
<p>SKILLS TO MAINTAIN</p> <p><i>Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.</i></p>	
<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> </ul> <p><i>Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.</i></p>	<p>N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p>N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.</p> <p>N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p>
<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> </ul> <p><i>Limit to linear expressions and to exponential expressions with integer exponents.</i></p>	<p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</p> <ol style="list-style-type: none"> <li>Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></li> </ol>
<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> </ul> <p><i>Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas with a linear focus.</i></p>	<p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>

## Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Represent and solve equations and inequalities graphically.</li> </ul> <p><i>For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where <math>f(x)</math> and <math>g(x)</math> are linear or exponential.</i></p>	<p>A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> <p>A.REI.11 Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*</p> <p>A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>
<ul style="list-style-type: none"> <li>Understand the concept of a function and use function notation.</li> </ul> <p><i>Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.</i></p> <p><i>Draw examples from linear and exponential functions. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.</i></p>	<p>F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p>F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p>F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p>

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> </ul> <p><i>For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.</i></p> <p><i>N.RN.1 and N.RN.2 will need to be referenced here before discussing exponential models with continuous domains.</i></p>	<p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>*</p> <p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i>*</p> <p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*</p>
<ul style="list-style-type: none"> <li>Analyze functions using different representations.</li> </ul> <p><i>For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as <math>y=3^n</math> and <math>y=100 \cdot 2^n</math>.</i></p>	<p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</p> <ul style="list-style-type: none"> <li>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</li> </ul> <p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>
<ul style="list-style-type: none"> <li>Build a function that models a relationship between two quantities.</li> </ul> <p><i>Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.</i></p>	<p>F.BF.1 Write a function that describes a relationship between two quantities.*</p> <ul style="list-style-type: none"> <li>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></li> </ul> <p>F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*</p>
<ul style="list-style-type: none"> <li>Build new functions from existing functions.</li> </ul> <p><i>Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.</i></p> <p><i>While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.</i></p>	<p>F.BF.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul> <p><i>For F.LE.3, limit to comparisons between exponential and linear models.</i></p>	<p>F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <ol style="list-style-type: none"> <li>Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.</li> <li>Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</li> <li>Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</li> </ol> <p>F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p> <p>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>
<ul style="list-style-type: none"> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul> <p><i>Limit exponential functions to those of the form <math>f(x) = b^x + k</math>.</i></p>	<p>F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.</p>

## Unit 3: Reasoning with Equations

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Unit 3: Reasoning with Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> </ul> <p><i>Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.</i></p>	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
<ul style="list-style-type: none"> <li>Solve equations and inequalities in one variable.</li> </ul> <p><i>Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as <math>5^x = 125</math> or <math>2^x = 1/16</math>.</i></p>	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
<ul style="list-style-type: none"> <li>Solve systems of equations.</li> </ul> <p><i>Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5, which requires students to prove the slope criteria for parallel lines.</i></p>	<p>A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p>A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p>

## Unit 4: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 4: Descriptive Statistics	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> </ul> <p><i>In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.</i></p>	<p>S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).</p> <p>S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</p> <p>S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</p>
<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> </ul> <p><i>Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.</i></p> <p><i>S.ID.6b should be focused on situations for which linear models are appropriate.</i></p>	<p>S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p>S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <ol style="list-style-type: none"> <li>Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</i></li> <li>Informally assess the fit of a function by plotting and analyzing residuals.</li> <li>Fit a linear function for scatter plots that suggest a linear association.</li> </ol>
<ul style="list-style-type: none"> <li>Interpret linear models.</li> </ul> <p><i>Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.</i></p>	<p>S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p> <p>S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.</p> <p>S.ID.9 Distinguish between correlation and causation.</p>



## Unit 5: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 5: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Experiment with transformations in the plane.</li> </ul> <p><i>Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.</i></p>	<p>G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</p> <p>G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).</p> <p>G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.</p> <p>G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.</p> <p>G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</p>
<ul style="list-style-type: none"> <li>Understand congruence in terms of rigid motions.</li> </ul> <p><i>Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.</i></p>	<p>G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</p> <p>G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</p> <p>G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</p>
<ul style="list-style-type: none"> <li>Make geometric constructions.</li> </ul> <p><i>Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects.</i></p> <p><i>Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.</i></p>	<p>G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). <i>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i></p> <p>G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</p>



## Unit 6: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean Theorem in 8<sup>th</sup> grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Unit 6: Connecting Algebra and Geometry Through Coordinates	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul> <p><i>This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.</i></p> <p><i>Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in Mathematics I involving systems of equations having no solution or infinitely many solutions.</i></p> <p><i>G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.</i></p>	<p>G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i></p> <p>G.GPE.5 Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</p> <p>G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*</p>